Scalar field perturbations in Horava-Lifshitz cosmology

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We study perturbations of a scalar field cosmology in Horava-Lifshitz gravity, adopting the most general setup without detailed balance but with the projectability condition. We derive the generalized Klein-Gordon equation, which is sixth-order in spatial derivatives. Then we investigate scalar field perturbations coupled to gravity in a flat Friedmann-Robertson-Walker background. In the sub-horizon regime, the metric and scalar field modes have independent oscillations with different frequencies and phases except in particular cases. On super-horizon scales, the perturbations become adiabatic during slow-roll inflation driven by a single field, and the comoving curvature perturbation is constant.

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I. INTRODUCTION

The background dynamics and the generation and evolution of perturbations during a period of inflation in the early universe, may deviate from the standard results if general relativity acquires significant ultra-violet (UV) corrections from a quantum gravity theory. Horava recently proposed such a theory [1], motivated by the Lifshitz theory in solid state physics [2]. Horava-Lifshitz (HL) theory has the interesting feature that it is non-relativistic in the UV regime, i.e., Lorentz invariance is broken. The effective speed of light in the theory diverges in the UV regime, which could potentially resolve the horizon problem without invoking inflation. Furthermore, scale-invariant super-horizon curvature perturbations could be produced without inflation [3–6]. Here we consider an HL model where primordial inflation does occur, and we investigate the changes which HL gravity induces in the dynamics and perturbations.

Horava assumed two conditions – detailed balance and projectability (though he also considered the case without detailed balance condition) [1]. So far most of the work on the HL theory has abandoned the projectability condition but maintained detailed balance [7–9]. One of the main reasons is that the resulting theory is much simpler to deal with, giving local rather than global energy constraints. However, breaking the projectability condition is problematic [10] and gives rise to an inconsistent theory [11]. With detailed balance, on the other hand, the scalar field is not UV stable [12], and the theory requires a non-zero negative cosmological constant and breaks parity in the purely gravitational sector [14] (see also [4]). To resolve these problems, various modifications have been proposed. The Sotiriou-Visser-Weinfurtner (SVW) [14] generalization is the most general setup of the HL theory with the projectability condition and without detailed balance. The preferred time that breaks Lorentz invariance leads to a reduced set of diffeomorphisms, and as a result, a spin-0 mode of the graviton appears. This mode is potentially dangerous and may cause strong coupling problems that could prevent the recovery of general relativity (GR) in the IR limit [10, 11, 15, 16]. To address this important issue and apply the theory to cosmology, two of the current authors studied linear cosmological perturbations of the Friedmann-Robertson-Walker (FRW) model with arbitrary spatial curvature in the SVW setup, and showed explicitly that the spin-0 scalar mode of the graviton is stable in both the IR and the UV regimes [20], provided that $0 \leq \xi \leq 2/3$, where ξ is a dynamical coupling parameter. This stability condition has the unwanted consequence that the scalar mode is a ghost [1, 15, 17, 18]. To tackle this problem, one may consider the theory in the range $\xi < 0$, where the sound speed $c_s^2 = \xi/(2-3\xi)$ is negative. However in the limit that the sound speed becomes small as $\xi \to 0$, one should undertake a non-linear analysis to determine whether the strong self-coupling of the scalar mode decouples [15], as in the Vainshtein mechanism in massive gravity [19].

In this paper we will be interested in studying cosmological perturbations in the SVW form of HL gravity as an example of a theory which explicitly breaks Lorentz invariance. We will investigate how standard results for linear perturbations in a scalar field cosmology are modified and how it may still be possible to recover some standard results in the long-wavelength or low-energy limit. We will not consider the non-linear perturbations and consider only the linear evolution of perturbations in a scalar field cosmology. We implicitly assume that the strong-coupling (or ghost) problem can be addressed via this mechanism or some other approach.

The general equations for perturbations of an FRW universe were derived in [20]. The coupling of matter to HL gravity has not been worked out yet in general, as now we no longer have the guiding principle of Lorentz invariance. Two exceptional cases are scalar and vector fields. Scalar fields were first investigated in [12] and [4]; the latter also studied vector fields and obtained the general couplings for both fields (see also [21]).

In Sec. II we obtain the stress 3-tensor for a scalar field in any spacetime, and then derive the generalized Klein-Gordon equation, which is sixth-order in space derivatives. In Sec. III we specialize to an FRW universe. We find that in the background, the generalized Klein-Gordon equation reduces to the standard general relativistic form, while the gravitational field equations have the Friedmann form after replacing the Newtonian constant G by $G/(1-3\xi/2)$. But the equations for linear perturbations are quite different, due to higher-order curvature terms. In particular, these terms lead to a gravitational effective anisotropic stress on small scales [20]. In Sec. IV, we study the curvature perturbation, showing that on large scales and in the adiabatic case, slow-roll leads to conservation of the curvature perturbation. We note that the large scale evolution of the curvature perturbation in the SVW setup was studied recently [22], and the conditions under which the curvature perturbation is conserved were discussed, but no specific matter fields were considered. In Sec. V we investigate the behavior of perturbations in the sub- and super-Hubble regimes. In Sec. VI, we study the coupled evolution of the adiabatic and entropy perturbations of the scalar field. We conclude in Sec. VII.

II. HL GRAVITY WITH PROJECTABILITY AND WITHOUT DETAILED BALANCE

In this section, we give a very brief introduction to HL gravity without detailed balance, but with the projectability condition. (For further details, see [14, 20].)

The dynamical variables are N, N^i and g_{ij}, in terms of which the metric takes the ADM form,

$$ds^2 = -N^2dt^2 + g_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right). \quad (2.1)$$

The projectability condition requires a homogeneous lapse function, $N = N(t)$. The total action has kinetic, potential and scalar field contributions:

$$S = \frac{1}{16\pi G}\int dt d^3x N\sqrt{g}\left(\mathcal{L}_K - \mathcal{L}_V + 16\pi G\mathcal{L}_M\right), \quad (2.2)$$

where

$$\begin{aligned}
\mathcal{L}_K &= K_{ij}K^{ij} - (1-\xi)K^2,\\
\mathcal{L}_V &= 2\Lambda - R + 16\pi G\left(g_2R^2 + g_3R_{ij}R^{ij}\right)\\
&\quad + \left(16\pi G\right)^2\left(g_4R^3 + g_5R\,R_{ij}R^{ij} + g_6R^i_jR^j_kR^k_i\right)\\
&\quad + \left(16\pi G\right)^2\left[g_7R\nabla^2R + g_8\left(\nabla_iR_{jk}\right)\left(\nabla^iR^{jk}\right)\right],\\
\mathcal{L}_M &= \frac{1}{2N^2}\left(\dot{\varphi} - N^i\nabla_i\varphi\right)^2 - \mathcal{V}\left(\varphi, \nabla_i\varphi, g_{ij}\right). \quad (2.3)
\end{aligned}$$

Here the covariant derivatives and Ricci and Riemann terms all refer to the three-metric g_{ij}, and K_{ij} is the extrinsic curvature, $K_{ij} = \left(-\dot{g}_{ij} + \nabla_iN_j + \nabla_jN_i\right)/2N$. The constants $\xi, g_I\,(I = 2,\dots 8)$ are coupling constants, and Λ is the cosmological constant. In the IR limit, all the higher-order curvature terms (with coefficients g_I) drop out, and the total action reduces when $\xi = 0$ to the Einstein-Hilbert action. The potential $\mathcal{V}(\varphi, \nabla_i\varphi, g_{ij}) = \mathcal{V}(\varphi, (\nabla\varphi)^2, \mathcal{P}_n)$ is defined by [4],

$$\begin{aligned}
\mathcal{V} &= V(\varphi) + \left[\frac{1}{2} + V_1(\varphi)\right](\nabla\varphi)^2 + V_2(\varphi)\mathcal{P}_1^2\\
&\quad + V_3(\varphi)\mathcal{P}_1^3 + V_4(\varphi)\mathcal{P}_2\\
&\quad + V_5(\varphi)(\nabla\varphi)^2\mathcal{P}_2 + V_6(\varphi)\mathcal{P}_1\mathcal{P}_2,\\
\mathcal{P}_n &\equiv \nabla^{2n}\varphi, \quad \nabla^2 \equiv g^{ij}\nabla_i\nabla_j, \quad (2.4)
\end{aligned}$$

where $V_s(\varphi)$ are arbitrary functions of φ only. In the GR limit, $V(\varphi)$ is the usual potential, and $V_s = 0$. In order to have the scalar field stable in the UV, we require that $V_6 < 0$. It should be noted that the potential (2.4) is slightly different from the one introduced in [4], but they differ only by boundary terms which do not affect the field equations.

Variation with respect to the lapse function $N(t)$ yields the Hamiltonian constraint,

$$\int d^3x\sqrt{g}\left(\mathcal{L}_K + \mathcal{L}_V\right) = 8\pi G\int d^3x\sqrt{g}\,J^t, \quad (2.5)$$

where

$$J^t = -2\left\{\frac{1}{2N^2}\left(\dot{\varphi} - N^i\nabla_i\varphi\right)^2 + \mathcal{V}\right\}. \quad (2.6)$$

Note that, unlike GR, there is no local Hamiltonian constraint. Variation with respect to the shift N^i yields the super-momentum constraint,

$$\nabla_j\pi^{ij} = 8\pi GJ^i, \quad (2.7)$$

where the super-momentum π^{ij} and matter current J^i are

$$\begin{aligned}
\pi^{ij} &\equiv \frac{\delta\mathcal{L}_K}{\delta\dot{g}_{ij}} = -K^{ij} + (1-\xi)Kg^{ij},\\
J^i &\equiv -N\frac{\delta\mathcal{L}_M}{\delta N_i} = \frac{1}{N}\left(\dot{\varphi} - N^k\nabla_k\varphi\right)\nabla_i\varphi. \quad (2.8)
\end{aligned}$$

The matter field satisfies the conservation laws [10, 20],

$$\begin{aligned}
&\int d^3x\sqrt{g}\left[\dot{g}_{kl}\tau^{kl} - \frac{1}{\sqrt{g}}\left(\sqrt{g}J^t\right)^{\cdot}\right.\\
&\qquad\left. + \frac{2N_k}{N\sqrt{g}}\left(\sqrt{g}J^k\right)^{\cdot}\right] = 0, \quad (2.9)\\
&\nabla^k\tau_{ik} - \frac{1}{N\sqrt{g}}\left(\sqrt{g}J_i\right)^{\cdot} - \frac{N_i}{N}\nabla_kJ^k\\
&\qquad - \frac{J^k}{N}\left(\nabla_kN_i - \nabla_iN_k\right) = 0. \quad (2.10)
\end{aligned}$$

Varying the action with respect to g_{ij} leads to the dynamical equations,

$$\frac{1}{N\sqrt{g}}\left(\sqrt{g}\pi^{ij}\right)^{\cdot} = -2K^i_kK^{kj} + 2(1-\xi)KK^{ij}$$

$$+\frac{1}{N}\nabla_k\left[N^k\pi^{ij}-2\pi^{k(i}\nabla_k N^{j)}\right]$$
$$+\frac{1}{2}\mathcal{L}_K g^{ij}+F^{ij}+8\pi G\tau^{ij}, \tag{2.11}$$

where F_{ij} is given in the Appendix.

The stress 3-tensor τ_{ij} for a scalar field is given by

$$\begin{aligned}\tau_{ij} \;\equiv\; & -\frac{2}{\sqrt{g}}\frac{\delta\left(\sqrt{g}\mathcal{L}_M\right)}{\delta g^{ij}}\\
&= \mathcal{L}_M g_{ij}+\left(\nabla_i\varphi\right)\left(\nabla_j\varphi\right)\left(1+2V_1+2V_5\mathcal{P}_2\right)\\
&+ g_{ij}\left(\nabla^2\varphi\right)\mathcal{V}_{,1}+\left(\nabla^k\mathcal{V}_{,1}\right)\left(\nabla_k\varphi\right)g_{ij}\\
&- 2\left(\nabla_{(i}\mathcal{V}_{,1}\right)\left(\nabla_{j)}\varphi\right)+g_{ij}\left(\nabla^4\varphi\right)\mathcal{V}_{,2}\\
&- 2\nabla_{(i}\nabla_k\left[\left(\nabla^k\mathcal{V}_{,2}\right)\left(\nabla_{j)}\varphi\right)\right]\\
&- 2\left(\nabla^k\mathcal{V}_{,2}\right)\left(\nabla_k\nabla_i\nabla_j\varphi\right)\\
&+ g_{ij}\left(\nabla^k\mathcal{V}_{,2}\right)\left(\nabla_k\nabla^2\varphi\right)\\
&- 2\left(\nabla_{(i}\mathcal{V}_{,2}\right)\left(\nabla_{j)}\nabla^2\varphi\right)\\
&+ 2\nabla_k\left[\left(\nabla^k\mathcal{V}_{,2}\right)\left(\nabla_i\nabla_j\varphi\right)\right]\\
&+ 2\nabla_{(i}\left[\left(\nabla^k\mathcal{V}_{,2}\right)\left(\nabla_k\nabla_{j)}\varphi\right)\right]\\
&- g_{ij}\nabla^k\left[\left(\nabla^l\mathcal{V}_{,2}\right)\left(\nabla_l\nabla_k\varphi\right)\right]\\
&+ g_{ij}\nabla_k\nabla_l\left[\left(\nabla^k\varphi\right)\left(\nabla^l\mathcal{V}_{,2}\right)\right].\end{aligned} \tag{2.12}$$

Variation of the total action with respect to φ yields the generalized Klein-Gordon equation,

$$\begin{aligned}\frac{1}{N\sqrt{g}}\left[\frac{\sqrt{g}}{N}\left(\dot\varphi-N^i\nabla_i\varphi\right)\right]^{\cdot} &= \nabla_i\left[\frac{N^i}{N^2}\left(\dot\varphi-N^k\nabla_k\varphi\right)\right]\\
&+\nabla^i\left[\nabla_i\varphi\left(1+2V_1+2V_5\mathcal{P}_2\right)\right]\\
&-\mathcal{V}_{,\varphi}-\nabla^2\left(\mathcal{V}_{,1}\right)-\nabla^4\left(\mathcal{V}_{,2}\right),\end{aligned} \tag{2.13}$$

where

$$\begin{aligned}\mathcal{V}_{,\varphi} &\equiv \frac{\partial\mathcal{V}}{\partial\varphi}=V'+V_1'(\nabla\varphi)^2+V_2'\mathcal{P}_1^2\\
&\quad +V_3'\mathcal{P}_1^3+V_4'\mathcal{P}_2+V_5'(\nabla\varphi)^2\mathcal{P}_2+V_6'\mathcal{P}_1\mathcal{P}_2,\\
\mathcal{V}_{,1} &\equiv \frac{\partial\mathcal{V}}{\partial\mathcal{P}_1}=2V_2\mathcal{P}_1+3V_3\mathcal{P}_1^2+V_6\mathcal{P}_2,\\
\mathcal{V}_{,2} &\equiv \frac{\partial\mathcal{V}}{\partial\mathcal{P}_2}=V_4+V_5(\nabla\varphi)^2+V_6\mathcal{P}_1.\end{aligned} \tag{2.14}$$

III. COSMOLOGICAL PERTURBATIONS IN A FLAT FRW BACKGROUND

For the homogeneous and isotropic FRW universe with scale factor $a(\eta)$ and conformal Hubble rate $\mathcal{H}=a'/a$, the gravitational field equations, coupled with a scalar field described in the last section, are given by Eqs. (B.1) and (B.2) in the appendix. In the flat case, they reduce to

$$\left(1-\frac{3}{2}\xi\right)\frac{\mathcal{H}^2}{a^2}=\frac{8\pi G}{3}\bar\rho_\varphi+\frac{\Lambda}{3}, \tag{3.1}$$

$$\left(1-\frac{3}{2}\xi\right)\frac{\mathcal{H}'}{a^2}=-\frac{4\pi G}{3}(\bar\rho_\varphi+3\bar p_\varphi)+\frac{1}{3}\Lambda, \tag{3.2}$$

where

$$\bar\rho_\varphi=\frac{1}{2a^2}\bar\varphi'^2+V(\bar\varphi),\quad \bar p_\varphi=\frac{1}{2a^2}\bar\varphi'^2-V(\bar\varphi), \tag{3.3}$$

From Eqs.(3.1) - (3.3), or directly from Eq. (2.13), we find

$$\bar\varphi''+2\mathcal{H}\bar\varphi'+a^2V'(\bar\varphi)=0, \tag{3.4}$$

thus recovering the standard Klein-Gordon equation. All the corrections due to high-order curvature terms vanish, and Eqs. (3.1)–(3.4) are identical to those in GR, with modified effective gravitational constant $G\to G_{eff}=G/(1-3\xi/2)$. Therefore, all the results obtained for scalar field cosmologies in GR for a spatially-flat FRW background are equally applicable to the spatially flat HL universe, including those for inflation, as far as only the homogeneous background is concerned. For example, the conditions for slow-roll inflation in the flat HL universe are $\epsilon_V, |\eta_V|\ll 1$, where

$$\epsilon_V\equiv\frac{1-3\xi/2}{16\pi G}\frac{V'^2}{V^2},\quad \eta_V\equiv\frac{1-3\xi/2}{8\pi G}\frac{V''}{V}. \tag{3.5}$$

However, inhomogeneous perturbations will be quite different, as the higher-order curvature corrections now have non-zero contributions. In the quasi-longitudinal gauge [20]

$$ds^2=a^2\left[-d\eta^2+2B_{,i}dx^id\eta+(1-2\psi)d\vec{x}^{\,2}\right], \tag{3.6}$$

we find that

$$\begin{aligned}J^t&=-2\left(\bar\rho_\varphi+\delta\rho_\varphi\right),\quad J_i=\partial_i q_\varphi,\\
\tau^i_j&=\frac{1}{a^2}\Big[\left(\bar p_\varphi+\delta p_\varphi+2\bar p_\varphi\psi\right)\delta^i_j\\
&\qquad+\left(\partial^i\partial_j-\frac{1}{3}\delta^i_j\nabla^2\right)\Pi_\varphi\Big],\end{aligned} \tag{3.7}$$

where

$$\begin{aligned}\delta\rho_\varphi &= \delta\rho_\varphi^{GR}+\frac{V_4}{a^4}\nabla^4\delta\varphi=\frac{\bar\varphi'}{a^2}\delta\varphi'+V'\delta\varphi+\frac{V_4}{a^4}\nabla^4\delta\varphi,\\
\delta p_\varphi &= \delta p_\varphi^{GR}=\frac{1}{a^2}\left(\bar\varphi'\delta\varphi'-a^2V'\delta\varphi\right),\\
q_\varphi &= q_\varphi^{GR}=\frac{\bar\varphi'}{a}\delta\varphi=-a(\bar\rho_\varphi+\bar p_\varphi)v_\varphi,\ v_\varphi=-\frac{\delta\varphi'}{\bar\varphi'}\\
\Pi_\varphi &= \Pi_\varphi^{GR}=0.\end{aligned} \tag{3.8}$$

The linearization of the generalized Klein-Gordon equation (2.13) yields

$$\begin{aligned}&\delta\varphi''+2\mathcal{H}\delta\varphi'+a^2V''\delta\varphi-\nabla^2\delta\varphi-\bar\varphi'\left(3\psi'+\nabla^2B\right)\\
&\quad=2\left(V_1-\frac{V_2+V_4'}{a^2}\nabla^2-\frac{V_6}{a^4}\nabla^4\right)\nabla^2\delta\varphi,\end{aligned} \tag{3.9}$$

where the deviations from GR are on the right, and are gradient terms, as expected. From Eqs. (B.4)–(B.8), we find that for a spatially flat background the linearized Hamiltonian constraint, energy conservation, trace dynamical equation, super-momentum constraint, and trace-free dynamical equation are, respectively,

$$\int d^3x \left[\nabla^2\psi - \left(1-\frac{3}{2}\xi\right)\mathcal{H}\left(\nabla^2 B + 3\psi'\right) - 4\pi G\left(\bar\varphi'\delta\varphi' + a^2 V'\delta\varphi + \frac{V_4}{a^2}\nabla^4\delta\varphi\right)\right] = 0, \quad (3.10)$$

$$\int d^3x a^2\bar\varphi'\left(\delta\varphi'' + 2\mathcal{H}\delta\varphi' + a^2 V''\delta\varphi - 3\bar\varphi'\psi'\right) = -\int d^3x\left[V_4\nabla^2\delta\varphi' + \left(V_4'\bar\varphi' - V_4\mathcal{H}\right)\nabla^4\delta\varphi\right], \quad (3.11)$$

$$\psi'' + 2\mathcal{H}\psi' - \frac{\xi}{(2-3\xi)}\left(1+\frac{\alpha_1}{a^2}\nabla^2 + \frac{\alpha_2}{a^4}\nabla^4\right)\nabla^2\psi = \frac{8\pi G}{(2-3\xi)}\left(\bar\varphi'\delta\varphi' - a^2V'\delta\varphi\right), \quad (3.12)$$

$$(2-3\xi)\psi' - \xi\nabla^2 B = 8\pi G\bar\varphi'\delta\varphi, \quad (3.13)$$

$$\left(a^2B\right)' = \left(a^2 + \alpha_1\nabla^2 + \frac{\alpha_2}{a^2}\nabla^4\right)\psi, \quad (3.14)$$

where the HL constants α_1, α_2 are defined by Eq. (B.11).

The (global) energy conservation law, Eq. (3.11), is satisfied automatically, provided that $\delta\varphi$ satisfies the generalized Klein-Gordon equation (3.9). Note also that Eq. (3.12) is not independent and can be obtained from Eqs. (3.9), (3.13) and (3.14). Therefore we are left with three independent equations (3.9), (3.13) and (3.14), and one constraint, Eq. (3.10), for the three unknowns, ψ, B and $\delta\varphi$.

Equation (3.14) can be written as [20],

$$\Phi - \Psi = \frac{1}{a^2}\left(\alpha_1 + \frac{\alpha_2}{a^2}\nabla^2\right)\nabla^2\psi, \quad (3.15)$$

where Φ and Ψ are the usual gauge-invariant metric perturbations [23], and in the quasi-longitudinal gauge [20] are given by

$$\Phi \equiv \mathcal{H}B + B', \quad \Psi \equiv \psi - \mathcal{H}B. \quad (3.16)$$

It follows from Eq. (3.15) that the higher-order curvature corrections in the HL theory effectively create an anisotropic stress [20], $\Pi^{HL} = -(8\pi G a^6)^{-1}(\alpha_1 a^2 + \alpha_2\nabla^2)\nabla^2\psi$. On large scales this is negligible, but on small scales it could produce significant deviations from GR.

In the GR limit, i.e., $\xi = 0 = V_s$, these equations reduce, respectively, to the corresponding equations given in GR [24].

IV. ENERGY CONSERVATION AND THE CURVATURE PERTURBATION

An important quantity is the gauge-invariant curvature perturbation on uniform-density hypersurfaces [24],

$$\zeta \equiv -\psi - \frac{\mathcal{H}}{\bar\rho'_\varphi}\delta\rho_\varphi, \quad (4.1)$$

which is constant on large scales for adiabatic perturbations in GR. In GR this follows directly from local energy conservation [25]. Moreover for a single scalar field in GR, the local Hamiltonian constraint equation requires the non-adiabatic pressure perturbation to vanish on super-Hubble scales [26]. In this section, we show that the curvature perturbation is also constant on large scales during slow-roll inflation in the HL theory, although the mechanism whereby this arises is quite different from the GR case.

The generalized Klein-Gordon equation (3.9) can be rewritten as a perturbed energy balance equation:

$$\delta\rho'_\varphi + 3\mathcal{H}(\delta\rho_\varphi + \delta p_\varphi) - (\bar\rho_\varphi + \bar p_\varphi)\left[3\psi' - \nabla^2(v_\varphi - B)\right] = (\bar\rho_\varphi + \bar p_\varphi)\,\delta Q^{HL}, \quad (4.2)$$

where the energy non-conservation δQ^{HL} is defined as

$$\delta Q^{HL} \equiv -\frac{V_4}{a^2\bar\varphi'^2}\nabla^4\delta\varphi' + \frac{1}{\bar\varphi'}\left[-2V_1 + \frac{2V_6}{a^4}\nabla^4 + \frac{1}{a^2}\left(2V_2 + V_4' + \mathcal{H}\frac{V_4}{\bar\varphi'}\right)\nabla^2\right]\nabla^2\delta\varphi. \quad (4.3)$$

In the GR limit $\delta Q^{HL} = 0$ and we recover the standard equation [24]. Non-zero terms on the right-hand-side represent the violation of local energy conservation. We see that in HL gravity δQ^{HL} is suppressed on large scales, but on small scales local energy conservation is violated by higher-order (Planck suppressed) terms.

The curvature perturbation (4.1) obeys the evolution equation,

$$\zeta' = -\frac{\mathcal{H}}{\bar\rho_\varphi + \bar p_\varphi}\delta p_{\varphi\,nad} - \frac{1}{3}\nabla^2\left(v_\varphi - B\right) + \frac{1}{3}\delta Q^{HL}. \quad (4.4)$$

The non-adiabatic pressure perturbation is

$$\delta p_{\varphi\,nad} \equiv \delta p_\varphi - \frac{\bar p'_\varphi}{\bar\rho'_\varphi}\delta\rho_\varphi = \delta p^{GR}_{\varphi\,nad} + \delta p^{HL}_{\varphi\,nad}, \quad (4.5)$$

where

$$\delta p^{GR}_{\varphi\,nad} \equiv \frac{2}{3a^2}\left(2 + \frac{\bar\varphi''}{\mathcal{H}\bar\varphi'}\right)\left[\bar\varphi'\delta\varphi' - \left(\bar\varphi'' - \mathcal{H}\bar\varphi'\right)\delta\varphi\right],$$

$$\delta p^{HL}_{\varphi\,nad} \equiv \left(1 + \frac{2\bar\varphi''}{\mathcal{H}\bar\varphi'}\right)\frac{V_4}{3a^4}\nabla^4\delta\varphi. \quad (4.6)$$

Thus the non-adiabatic pressure perturbation has a contribution of the same form as in GR, and a specific HL contribution, which is negligible on large scales, but significant on small scales. In GR we can use the Hamiltonian constraint to show that $\delta p^{GR}_{\varphi\, nad}$ must vanish for a single field on large scales (if the curvature perturbation Ψ remains finite). In HL gravity of SVW form, we no longer have a local Hamiltonian constraint. However, in the case of slow-roll inflation (or any overdamped solution for a single scalar field) the existence of a unique attractor solution for the scalar field ensures that the local proper time derivative of the scalar field becomes a unique function of the local field, $\dot{\varphi} = f(\varphi)$. If the scalar field perturbations approach the same slow-roll attractor on large scales, then $\delta\dot{\varphi} = f'(\varphi)\delta\varphi = (\ddot{\varphi}/\dot{\varphi})\delta\varphi$. It follows from Eq. (4.6) that $\delta p^{GR}_{\varphi\, nad} = 0$. Since the HL corrections are also negligible on large scales, we expect the perturbations to be adiabatic in the super-horizon region during single-field slow-roll inflation, as in GR.

Therefore, the curvature perturbation on uniform density hypersurfaces will be constant on superhorizon scales for slow-roll inflation, which is the same as we obtain in GR. This is expected, because the difference between GR and the HL theory is principally in the UV regime, where the higher-order curvature corrections become important. On large scales, these corrections are negligible, and we expect that both of them will give the same results. However, the mechanism here is quite different. In GR, it is energy conservation that ensures $\delta p^{GR}_{\varphi\, nad} = 0$ on large scales [24], while here it is the slow-roll conditions that give $\delta p^{GR}_{\varphi\, nad} \simeq 0$. In the HL theory, the (local) conservation law of GR is replaced by its integral form, Eq. (3.11). This indicates that more generally the perturbations need not be adiabatic and that the curvature perturbation ζ may not be constant on superhorizon scales in the HL cosmology in the absence of slow-roll (see also the general discussion in [22]).

V. SUB- AND SUPER-HORIZON PERTURBATIONS

Working in Fourier space, and defining

$$u_k = a\delta\varphi_k, \quad \chi_k = a\psi_k, \tag{5.1}$$

Equations (3.9), (3.12), (3.13) and (3.14) lead to

$$\chi_k' - \mathcal{H}\chi_k = \frac{8\pi G}{2-3\xi}\bar{\varphi}' u_k - \frac{\xi a k^2}{2-3\xi}B_k, \tag{5.2}$$

$$B_k' + 2\mathcal{H}B_k = \frac{1}{a}\left(1 - \frac{\alpha_1}{a^2}k^2 + \frac{\alpha_2}{a^4}k^4\right)\chi_k, \tag{5.3}$$

$$u_k'' + \left(\omega_\varphi^2 - \frac{a''}{a}\right)u_k = \bar{\varphi}'\Big[3\big(\chi_k' - \mathcal{H}\chi_k\big) - k^2 a B_k\Big], \tag{5.4}$$

$$\chi_k'' + \left(\omega_\psi^2 - \frac{a''}{a}\right)\chi_k = \frac{8\pi G}{2-3\xi}\Big[\bar{\varphi}' u_k' - \big(\mathcal{H}\bar{\varphi}' + a^2 V'\big)u_k\Big], \tag{5.5}$$

where

$$\omega_\varphi^2 = a^2 V'' + k^2\left(1 + 2V_1 + \frac{2(V_2 + V_4')}{a^2}k^2 - \frac{2V_6}{a^4}k^4\right),$$
$$\omega_\psi^2 = \frac{\xi k^2}{2-3\xi}\left(1 - \frac{\alpha_1}{a^2}k^2 + \frac{\alpha_2}{a^4}k^4\right). \tag{5.6}$$

From Eqs. (5.4) and (5.6) we can see that in order for the scalar field to be stable in the UV regime, we require that $V_6 < 0$. Similarly, the metric perturbation ψ is UV stable if $\xi\alpha_2/(2-3\xi) \geq 0$.

To study the above equations further, we consider them in the sub-horizon and super-horizon regimes separately.

A. Sub-horizon scales

On sub-horizon scales and for sufficiently large k^2 the highest-order curvature terms dominate the dynamics. From Eq. (5.6), assuming $V_6 \neq 0$ and $\xi\alpha_2 \neq 0$, we have

$$\omega_\varphi^2 \simeq -\frac{2V_6}{a^4}\,k^6, \quad \omega_\psi^2 \simeq \frac{\xi\alpha_2}{(2-3\xi)a^4}\,k^6, \tag{5.7}$$

and then from Eqs. (5.4) and (5.5) have the oscillating solutions (for $\xi \neq 2/3$),

$$u_k \simeq \frac{u_0}{\sqrt{\omega_\varphi}}e^{i\omega_\varphi \eta}, \quad \chi_k \simeq \frac{\chi_0}{\sqrt{\omega_\psi}}e^{i\omega_\psi \eta}, \tag{5.8}$$

where u_0 and χ_0 are constants. As noticed by [3], the dispersion relationship (5.7) yields scale-invariant primordial perturbations. From Eqs. (5.4)–(5.6) one can see that the scale-invariance is not exact [6], due to the low-energy corrections and the coupling to metric perturbations.

In the UV regime, the scalar field mode u_k and the metric perturbation mode χ_k are oscillating independently, although the two metric perturbation modes χ_k and B_k are oscillating with the same frequency but a different constant phase:

$$B_k \simeq -i\chi_0\alpha_2\left(\frac{2-3\xi}{\xi\alpha_2}\right)^{1/2}\frac{k}{a^3\sqrt{\omega_\psi}}e^{i\omega_\psi\eta}, \tag{5.9}$$

which follows from Eq. (5.2).

When $\xi = 0$ (which corresponds to the limit of GR in the IR regime), or $\xi = 2/3$ (when the theory has an additional symmetry, the anisotropic Weyl invariance [1]), u_k is still given by Eq. (5.8), but the metric modes oscillate with same frequency, ω_φ, and so are coupled to the scalar field mode.

B. Super-horizon scales

When $k \ll \mathcal{H}$ then, up to order k^2, Eqs. (5.2)–(5.4) become

$$\chi_k' - \mathcal{H}\chi_k = \frac{8\pi G}{2-3\xi}\bar{\varphi}' u_k - \frac{\xi a k^2}{2-3\xi}B_k, \tag{5.10}$$

$$B_k' + 2\mathcal{H}B_k = \frac{1}{a}\left(1 - \frac{\alpha_1}{a^2}k^2\right)\chi_k, \quad (5.11)$$

$$u_k'' + \left(\omega_\varphi^2 - \frac{a''}{a}\right)u_k = \bar{\varphi}'\Big[3\big(\chi_k' - \mathcal{H}\chi_k\big) - k^2 a B_k\Big], \quad (5.12)$$

$$\chi_k'' + \left(\omega_\psi^2 - \frac{a''}{a}\right)\chi_k = \frac{8\pi G}{2-3\xi}\Big[\bar{\varphi}' u_k' - \big(\mathcal{H}\bar{\varphi}' + a^2 V'\big)u_k\Big], \quad (5.13)$$

where, to order k^2, we have from Eq. (5.6)

$$\omega_\varphi^2 \simeq a^2 V'' + \big(1 + 2V_1\big)k^2, \;\; \omega_\psi^2 \simeq \frac{\xi}{2-3\xi}k^2. \quad (5.14)$$

To zeroth order in k^2, from Eqs. (5.10) and (5.12) one can obtain an equation that only involves u_k. Once this equation is solved, from Eqs. (5.10) and (5.11) one can find the metric perturbations χ_k and B_k by quadrature. To order k^2 Eqs. (5.10)–(5.12) reduce to those in GR [24] if $\alpha_1 \to 0$ and $V_1 \to 0$ with $G \to G_{eff}$ if $\xi \neq 2/3$, but without a local Hamiltonian constraint equation.

In the extreme slow-roll (de Sitter) limit, we take $\bar{\varphi}' \simeq 0 \simeq V'$, and $a \simeq -(H\eta)^{-1}$ (with H constant),

$$\omega_\varphi^2 - \frac{a''}{a} \;=\; \big(1 + 2V_1\big)k^2 - \left(1 - \frac{3\eta_V}{2-3\xi}\right)\frac{2}{\eta^2}. \quad (5.15)$$

If in addition we take the massless limit, $\eta_V \simeq 0$, Eq. (5.12) has the solution to order k^2

$$\begin{aligned} u_k \;&=\; -\frac{C_\varphi}{H\eta}\left[1 + \frac{1}{2}\left(1 + 2V_1\right)k^2\eta^2\right] \\ &\quad + D_\varphi \eta^2\left[1 - \frac{1}{10}(1 + 2V_1)k^2\eta^2\right]. \quad (5.16) \end{aligned}$$

The first term represents the growing mode which corresponds to a constant scalar field perturbation on large scales, $\delta\varphi_k \to C_\varphi$ as $k\eta \to 0$, while the second term is the decaying mode.

As in GR the scalar field perturbations decouple from the metric perturbations in the de Sitter limit. In GR this is because the local constraint equations require gauge-invariant scalar metric perturbations to vanish in this limit, but in HL gravity the metric has independent scalar perturbations. Integrating Eq. (5.13) we obtain

$$\begin{aligned} \chi_k \;&\simeq\; -\frac{C_\chi}{H\eta}\left[1 + \frac{1}{2}\left(\frac{\xi k^2}{2-3\xi}\right)\eta^2\right] \\ &\quad + \frac{D_\chi}{3H}\left(\frac{\xi k^2}{2-3\xi}\right)\eta^2\left[1 - \frac{1}{10}\left(\frac{\xi k^2}{2-3\xi}\right)\eta^2\right]. \quad (5.17) \end{aligned}$$

Then, from Eqs. (5.1), (5.10) and (5.11) we find that

$$\begin{aligned} \psi_k \;&\simeq\; C_\chi\left[1 + \frac{1}{2}\left(\frac{\xi k^2}{2-3\xi}\right)\eta^2\right], \\ &\quad - \frac{D_\chi}{3}\left(\frac{\xi k^2}{2-3\xi}\right)\eta^3\left[1 - \frac{1}{10}\left(\frac{\xi k^2}{2-3\xi}\right)\eta^2\right], \\ B_k \;&\simeq\; -C_\chi\eta\left[1 - \frac{1}{2}\left(\frac{\xi}{2-3\xi} - 2\alpha_1 H^2\right)k^2\eta^2\right] \\ &\quad + D_\chi\eta^2\left[1 - \frac{1}{6}\left(\frac{\xi k^2}{2-3\xi}\right)\eta^2\right]. \quad (5.18) \end{aligned}$$

On large scales we have $\psi_k = -B_k/\eta \to C_\chi$ as $k\eta \to 0$, but this corresponds to a gauge mode. In terms of the gauge-invariant quantities (3.16) we find

$$\begin{aligned} \Phi_k \;&\simeq\; C_\chi\left[\frac{\xi}{2-3\xi} - 2\alpha_1 H^2\right]k^2\eta^2 \\ &\quad + D_\chi\eta\left[1 - \frac{1}{2}\left(\frac{\xi k^2}{2-3\xi}\right)\eta^2\right], \\ \Psi_k \;&\simeq\; C_\chi\left[\frac{\xi}{2-3\xi} - \alpha_1 H^2\right]k^2\eta^2 \\ &\quad + D_\chi\eta\left[1 - \frac{1}{2}\left(\frac{\xi k^2}{2-3\xi}\right)\eta^2\right], \quad (5.19) \end{aligned}$$

from which we find that $\Phi_k - \Psi_k \simeq -C_\chi\alpha_1 H^2 k^2\eta^2$.

Thus although the gauge invariant metric perturbations Φ and Ψ are not constrained to vanish in the slow-roll limit, their dynamical evolution leads to $\Phi = \Psi \to 0$ at late times ($\eta \to 0$). Similarly, although the HL theory does lead to an effective anisotropic stress (3.15), this is of order k^2 and vanishes in the large-scale limit.

VI. COUPLED ADIABATIC AND ENTROPY PERTURBATIONS ON LARGE SCALES

The gauge-invariant variable ζ is closely related to the comoving curvature perturbation for scalar field perturbations [23, 24]

$$\mathcal{R} \equiv \psi + \frac{\mathcal{H}}{\bar{\varphi}'}\delta\varphi = -\zeta + \mathcal{H}\left(\frac{\delta\varphi}{\varphi'} - \frac{\delta\rho_\varphi}{\bar{\rho}_\varphi'}\right). \quad (6.1)$$

The two variables coincide, up to a choice of sign, for adiabatic perturbations. Thus the comoving curvature perturbation should also be conserved on large scales for adiabatic scalar field perturbations. From the definition of the comoving curvature perturbation it is straightforward to derive

$$\mathcal{R}' = \mathcal{H}\mathcal{S} + \psi' + \frac{\mathcal{H}' - \mathcal{H}^2}{\bar{\varphi}'}\delta\varphi\,, \quad (6.2)$$

where we define the dimensionless intrinsic entropy perturbation for the field

$$\mathcal{S} \equiv \frac{\delta\varphi'}{\bar{\varphi}'} - \frac{(\bar{\varphi}'' - \mathcal{H}\bar{\varphi}')}{\bar{\varphi}'^2}\delta\varphi\,. \quad (6.3)$$

Note that the GR non-adiabatic pressure perturbation in Eq. (4.6) is given by

$$\delta p^{GR}_{\varphi\, nad} = -\frac{2\bar{\varphi}' V'}{3\mathcal{H}}\mathcal{S}. \quad (6.4)$$

Using the HL super-momentum constraint (3.13),

$$\mathcal{R}' = \mathcal{H}\mathcal{S} + \frac{\xi}{2-3\xi}\nabla^2 B\,. \tag{6.5}$$

In the GR limit, when $\xi = 0$, this reduces to $\mathcal{R}' = \mathcal{H}\mathcal{S}$ on all scales.

Using the generalized Klein-Gordon equation (3.9), we obtain a first-order equation for $\mathcal{S}$ on large scales

$$\begin{aligned}\mathcal{S}' + \left(2\frac{\bar{\varphi}''}{\bar{\varphi}'} + \mathcal{H}\right)\mathcal{S} \; &= \; \frac{(1+2V_1)}{\bar{\varphi}'}\nabla^2\delta\varphi \\ &\quad + \frac{2}{2-3\xi}\nabla^2 B + \mathcal{O}(\nabla^4),\end{aligned} \tag{6.6}$$

where $\mathcal{O}(\nabla^4)$ denotes Planck-suppressed higher-order terms. In slow-roll, and neglecting spatial gradients on large scales, we find

$$\mathcal{S}' + 3\mathcal{H}\mathcal{S} \simeq 0, \quad \mathcal{R}'' + 2\mathcal{H}\mathcal{R}' \simeq 0. \tag{6.7}$$

Thus we find a constant mode and a rapidly decaying mode on large scales

$$\mathcal{R} \simeq C + D\int\frac{d\eta}{a^2}\,. \tag{6.8}$$

This is the same slow-roll expression as is found in GR, and is consistent with our earlier result that ζ is conserved for adiabatic perturbations on large scales.

However, again we see that the derivation is rather different from GR. The local Hamiltonian constraint in GR enforces adiabaticity on large scales [24]

$$\mathcal{S} = \frac{1}{4\pi G\bar{\varphi}'^2}\nabla^2\Psi. \tag{6.9}$$

In the HL case we have no such local constraint, but slow-roll evolution (6.7) leads to rapidly decaying entropy perturbations at late times.

Finally, we note that in the study of perturbations for a single scalar field in GR, the gauge-invariant field perturbation [24], $\delta\varphi_f = \delta\varphi + \bar{\varphi}'\psi/\mathcal{H}$, is often used. The Klein-Gordon equation can be cast in a form that involves only $\delta\varphi_f$ [27]. We find that this becomes impossible in HL gravity for two reasons. (a) In GR, the super-Hamiltonian constraint is used to eliminate the metric perturbations. However, in HL theory, the constraint is replaced by an integral form (3.10), which cannot be used in the same way. (b) Higher-order curvature corrections enter the field equations, and these terms vanish only on super-horizon scales. In terms of $\delta\varphi_f$, the generalized Klein-Gordon equation (3.9) is given in an Appendix.

VII. CONCLUSIONS

We have studied perturbations of a scalar field cosmology in Horava-Lifshitz gravity with projectability and without detailed balance. After giving the field equations for an arbitrary spacetime in Sec. II, including the generalized Klein-Gordon equation (which is sixth-order in spatial derivatives), we investigated linear perturbations about a flat FRW universe. In the flat FRW background, the field equations and generalized Klein-Gordon equation reduce to those in GR (under $G \to G_{eff}$). As a result, all the usual results regarding scalar field dynamics and slow-roll inflation in the flat FRW background also hold in the HL theory. However, the linear perturbations are quite different, due to the higher-order curvature terms in the effective action which enter the equations as higher order spatial derivatives. In addition, the Hamiltonian constraint and the conservation of energy now take integral forms.

In Sec. IV, we considered the evolution of ζ, the curvature perturbation on uniform-density hypersurfaces, which is conserved for adiabatic perturbations on large scales in GR. We identified the non-adiabatic pressure perturbation, which generalizes the expression in GR via a higher-order gradient correction. On large scales, the correction vanishes, while GR part vanishes due to the slow-roll conditions. Therefore, similar to GR, super-horizon curvature perturbations are adiabatic and conserved (for the curvature perturbation on uniform density hypersurfaces and the comoving curvature perturbation). However, the mechanism for conservation is different from GR. In GR, it is the local Hamiltonian constraint that enforces $\delta p^{GR}_{\varphi\, nad} \simeq 0$ on large scales, while here it is the slow-roll dynamics. In the HL theory, the conservation law of GR is replaced by its integral form. This indicates that in more general cases than slow-roll, the scalar field perturbations need not be adiabatic on large scales, and consequently the curvature perturbation need not be constant. This is an aspect of HL cosmology that deserves further investigation.

In Sec. V, we investigated the perturbations in the sub- and super-horizon limits. In the UV sub-horizon limit, the dispersion relations for scalar field and metric modes is of the form $\omega^2 \propto k^6$, and it has been argued that this can lead to scale-invariant primordial perturbations [15]. We identified the low-energy corrections to exact scale-invariance. The UV metric and scalar field modes oscillate independently with different frequencies and phases, except for the two special cases $\xi = 0$ and $\xi = 2/3$. At these two fixed points, they are oscillating with the same frequency, although still with different phases. In the IR super-horizon limit, the coupled equations reduce to a single second-order equation, and we solved for the gauge-invariant metric potentials in the de Sitter limit. In Sec. VI we showed, using the coupled adiabatic and entropy perturbations, how a constant curvature perturbation is recovered on large scales in slow-roll inflation.

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Appendix A: The F_{ij} tensor

The F_{ij} tensor in Eq. (2.11) is defined in an arbitrary spacetime as

$$F^{ij} \equiv \frac{1}{\sqrt{g}}\frac{\delta\left(-\sqrt{g}\mathcal{L}_V\right)}{\delta g_{ij}} = \sum_{s=0}^{8}\frac{g_s}{(16\pi G)^{n_s/2}}\left(F_s\right)^{ij}, \quad \text{(A.1)}$$

where the additional constants are given by $g_0 = 32\pi G\Lambda$, $g_1 = -1$, and $n_s = (2, 0, -2, -2, -4, -4, -4, -4, -4)$. The geometric 3-tensors $(F_s)_{ij}$ are:

$$\begin{aligned}
(F_0)_{ij} &= -\frac{1}{2}g_{ij},\\
(F_1)_{ij} &= R_{ij} - \frac{1}{2}Rg_{ij},\\
(F_2)_{ij} &= 2\left(R_{ij} - \nabla_i\nabla_j\right)R - \frac{1}{2}g_{ij}\left(R - 4\nabla^2\right)R,\\
(F_3)_{ij} &= \nabla^2 R_{ij} - \left(\nabla_i\nabla_j - 3R_{ij}\right)R - 4\left(R^2\right)_{ij}\\
&\quad + \frac{1}{2}g_{ij}\left(3R_{kl}R^{kl} + \nabla^2 R - 2R^2\right),\\
(F_4)_{ij} &= 3\left(R_{ij} - \nabla_i\nabla_j\right)R^2 - \frac{1}{2}g_{ij}\left(R - 6\nabla^2\right)R^2,\\
(F_5)_{ij} &= \left(R_{ij} + \nabla_i\nabla_j\right)\left(R_{kl}R^{kl}\right) + 2R\left(R^2\right)_{ij}\\
&\quad + \nabla^2\left(RR_{ij}\right) - \nabla^k\left[\nabla_i\left(RR_{jk}\right) + \nabla_j\left(RR_{ik}\right)\right]\\
&\quad - \frac{1}{2}g_{ij}\left[\left(R - 2\nabla^2\right)\left(R_{kl}R^{kl}\right)\right.\\
&\quad \left. - 2\nabla_k\nabla_l\left(RR^{kl}\right)\right],\\
(F_6)_{ij} &= 3\left(R^3\right)_{ij} + \frac{3}{2}\left[\nabla^2\left(R^2\right)_{ij}\right.\\
&\quad \left. - \nabla^k\left(\nabla_i\left(R^2\right)_{jk} + \nabla_j\left(R^2\right)_{ik}\right)\right]\\
&\quad - \frac{1}{2}g_{ij}\left[R^k_l R^l_m R^m_k - 3\nabla_k\nabla_l\left(R^2\right)^{kl}\right],\\
(F_7)_{ij} &= 2\nabla_i\nabla_j\left(\nabla^2 R\right) - 2\left(\nabla^2 R\right)R_{ij}\\
&\quad + \left(\nabla_i R\right)\left(\nabla_j R\right) - \frac{1}{2}g_{ij}\left[\left(\nabla R\right)^2 + 4\nabla^4 R\right],\\
(F_8)_{ij} &= \nabla^4 R_{ij} - \nabla_k\left(\nabla_i\nabla^2 R^k_j + \nabla_j\nabla^2 R^k_i\right)\\
&\quad - \left(\nabla_i R^k_l\right)\left(\nabla_j R^l_k\right) - 2\left(\nabla^k R^l_i\right)\left(\nabla_k R_{jl}\right)\\
&\quad - \frac{1}{2}g_{ij}\left[\left(\nabla_k R_{lm}\right)^2 - 2\left(\nabla_k\nabla_l\nabla^2 R^{kl}\right)\right]. \quad \text{(A.2)}
\end{aligned}$$

Appendix B: Cosmological Perturbations in an FRW Background

We summarize the key cosmological perturbation equations for the FRW metric, $ds^2 = a^2(-d\eta^2 + \gamma_{ij}dx^i dx^j)$, where $\gamma_{ij} = [1 + K(x^2 + y^2 + z^2)/4]^{-2}\delta_{ij}$, with $K = 0, \pm 1$. In the background, the Hamiltonian constraint (2.5) and dynamical equation (2.11) reduce to [14, 20],

$$\begin{aligned}
\left(1 - \frac{3}{2}\xi\right)\frac{\mathcal{H}^2}{a^2} + \frac{K}{a^2} &= \frac{8\pi G}{3}\bar{\rho}_\varphi + \frac{\Lambda}{3}\\
&\quad + \frac{2\beta_1 K^2}{a^4} + \frac{4\beta_2 K^3}{a^6}, \quad \text{(B.1)}\\
\left(1 - \frac{3}{2}\xi\right)\frac{\mathcal{H}'}{a^2} &= -\frac{4\pi G}{3}\left(\bar{\rho}_\varphi + 3\bar{p}_\varphi\right) + \frac{1}{3}\Lambda\\
&\quad - \frac{2\beta_1 K^2}{a^4} - \frac{8\beta_2 K^3}{a^6}, \quad \text{(B.2)}
\end{aligned}$$

where

$$\beta_1 = 16\pi G(3g_2 + g_3),\ \beta_2 = (16\pi G)^2(9g_4 + 3g_5 + g_6). \quad \text{(B.3)}$$

Then to first-order, using [20], the Hamiltonian and super-momentum constraints, the trace and trace-free dynamical equations, and energy conservation are given, respectively, by

$$\begin{aligned}
&\int \sqrt{\gamma}d^3x\Bigg[\left(\nabla^2 + 3K\right)\psi - \frac{\mathcal{H}(2 - 3\xi)}{2}\left(\nabla^2 B + 3\psi'\right)\\
&\quad - 2K\left(\frac{2\beta_1}{a^2} + \frac{6\beta_2 K}{a^4} + \frac{3g_7}{\zeta^4 a^4}\nabla^2\right)\left(\nabla^2 + 3K\right)\psi\\
&\quad - 4\pi G a^2\left(\frac{\bar{\varphi}'}{a^2}\delta\varphi' + V'\delta\varphi + \frac{V_4}{a^4}\nabla^4\delta\varphi\right)\Bigg] = 0, \quad \text{(B.4)}
\end{aligned}$$

$$(2 - 3\xi)\psi' - 2KB - \xi\nabla^2 B = 8\pi G\bar{\varphi}'\delta\varphi, \quad \text{(B.5)}$$

$$\begin{aligned}
&\psi'' + 2\mathcal{H}\psi' - \mathcal{F}\psi + \frac{1}{3}\left(\nabla^2 B' + 2\mathcal{H}\nabla^2 B\right)\\
&\quad - \frac{\gamma^{ij}\delta F_{ij}}{3(2 - 3\xi)} = \frac{8\pi G}{(2 - 3\xi)}\left(\bar{\varphi}'\delta\varphi' - a^2 V'\delta\varphi\right), \quad \text{(B.6)}
\end{aligned}$$

$$(B' + 2\mathcal{H}B)_{|\langle ij\rangle} + \delta F_{\langle ij\rangle} = 0, \quad \text{(B.7)}$$

$$\begin{aligned}
&\int d^3x\left(\delta\varphi'' + 2\mathcal{H}\delta\varphi' + a^2 V''\delta\varphi - 3\bar{\varphi}'\psi'\right)a^2\bar{\varphi}'\\
&\quad = -\int d^3x\left[V_4\vec{\nabla}^2\delta\varphi' + \left(V_4'\bar{\varphi}' - V_4\mathcal{H}\right)\vec{\nabla}^4\delta\varphi\right], \quad \text{(B.8)}
\end{aligned}$$

where a vertical bar denotes the covariant derivative with respect to γ_{ij}, and angled brackets on indices denote the symmetric trace-free part. Here

$$\mathcal{F} = \frac{2a^2}{(2 - 3\xi)}\left(-\Lambda + \frac{K}{a^2} + \frac{2\beta_1 K^2}{a^4} + \frac{12\beta_2 K^3}{a^6}\right), \quad \text{(B.9)}$$

and δF_{ij} is given by Eq. (A.1) in [20]. When $K = 0$, using [20],

$$\begin{aligned}
\delta F_{ij} &= 2\Lambda a^2\psi\delta_{ij}\\
&\quad - \left(1 + \frac{\alpha_1}{a^2}\nabla^2 + \frac{\alpha_2}{a^4}\nabla^4\right)\left(\partial_i\partial_j - \delta_{ij}\nabla^2\right)\psi, \quad \text{(B.10)}
\end{aligned}$$

where

$$\alpha_1 = 16\pi G(8g_2 + 3g_3),\ \alpha_2 = (16\pi G)^2(3g_8 - 8g_7). \quad \text{(B.11)}$$

Appendix C: Generalized Klein-Gordon equation in $\delta\varphi_f$

Using Eqs. (3.12) and (3.13),

$$\begin{aligned}
&\delta\varphi_f'' + 2\mathcal{H}\delta\varphi_f' - (1+2V_1)\nabla^2\delta\varphi_f + a^2V''\delta\varphi_f \\
&\quad + \frac{\mathcal{H}^2-\mathcal{H}'}{\bar\varphi'\mathcal{H}^2}\Big\{\big[2\bar\varphi'\left(\mathcal{H}'+\mathcal{H}^2\right)+3a^2\mathcal{H}V'\big]\delta\varphi_f \\
&\qquad\qquad + \mathcal{H}\bar\varphi'\delta\varphi_f'\Big\} \\
&\quad + \frac{2}{a^2}\Big[\,(V_2+V_4')+\frac{V_6}{a^2}\nabla^2\Big]\nabla^4\delta\varphi_f \\
&= \frac{\bar\varphi'}{(2-3\xi)\mathcal{H}}\Big\{\big[-2+4\xi-2(2-3\xi)V_1\big] \\
&\quad + \frac{1}{a^2}\big[\xi\alpha_1+2(2-3\xi)\left(V_2+V_4'\right)\big]\nabla^2 \\
&\quad + \frac{1}{a^4}\big[\xi\alpha_2+2(2-3\xi)V_6\big]\nabla^4\Big\}\nabla^2\psi \\
&\quad + \frac{1}{\mathcal{H}^2}\big[\bar\varphi'\left(4\mathcal{H}^3-2\mathcal{H}\mathcal{H}'-\mathcal{H}''\right) \\
&\qquad + 2a^2\left(\mathcal{H}^2-\mathcal{H}'\right)V'\big]\psi \\
&\quad + \frac{1}{(2-3\xi)\mathcal{H}^2}\big[(2-5\xi)\bar\varphi\mathcal{H}^2 \\
&\qquad - \xi\left(\bar\varphi\mathcal{H}'+2a^2\bar\varphi\mathcal{H}V'\right)\big]\nabla^2B, \qquad \text{(C.1)}
\end{aligned}$$

where the gauge-invariant variable $\delta\varphi_{flat}$ is defined as [24],

$$\delta\varphi_{flat} = \delta\varphi + \frac{\bar\varphi'}{\mathcal{H}}\psi. \qquad \text{(C.2)}$$

As noted above, and unlike the case of GR, the metric variables ψ and B remain in the equation.

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